MID-SEMESTER EXAMINATION
B. MATH III YEAR, I SEMESTER 2009-2010
COMPLEX ANALYSIS

Max. Score:100 Time limit: 3hrs.

1. Prove or disprove the following:
if f is an entire function and $g(z) = (f(z^-))^-$ (where a^- is the complex conjugate of a) then g is also an entire function. [15]

2. Find all entire functions f such that $[f(z)]^3 = e^z$ for all $z \in \mathbb{C}$. [10]

3. If $f(z) = \frac{z}{1+z}$ find $f(U)$. Is f a conformal equivalence of U onto $f(U)$?
Hint: use properties of Mobius transformations. [15]

4. Let γ be a continuously differentiable map from $[0,1]$ into $\mathbb{C}$ with $\gamma(0) = 1$ and $\gamma(1) = i$. Evaluate $\int\limits_\gamma (23 - 3z^5 + 7z^6 + 200z^{100})dz$. [15]

5. Prove that if p is a non-constant polynomial of degree n then $\{z : |p(z)| < 1\}$ is a bounded open set with atmost n connected components. Give an example to show that the number of components can be less than n. [15]

6. If f is an entire function such that $|f(z)| \geq |z|$ for all z prove that f is necessarily a polynomial. [10]

7. Let $f \in H(\Omega)$ and $f(z) \notin (-\infty, 0]$ for all $z \in \Omega$. Prove that $\log|f|$ is a harmonic function on Ω. Also prove that the conclusion is true for any $f \in H(\Omega)$ such that $f(z) \neq 0$ for all $z \in \Omega$. [20]